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The algorithm for generation of structured grids in deformed volumes of revolution

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The algorithm for generation of structured grids in deformed volumes of revolution

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Abstract. For the volume of revolution deformed by another volume of revolution, a grid generation algorithm is suggested. The algorithm is designed for multimaterial hydrodynamic simulation and for solving other physical and engineering problems. The algorithm represents the non-stationary procedure generating three-dimensional structured grids in domains with moving boundaries. The algorithm is developed within the variational approach for constructing optimal curvilinear grids. The volume of revolution is obtained by the rotation about the axis through 180° of a plane generatrix curve consisting of straight line segments and arcs of circles. The non-stationary algorithm is the iterative process at each stage of which the deformation of a grid and then its optimization are carried out. The deformation of a grid is implemented within the geometrical approach, and the optimization of a grid within the variational approach. Iterations are continued until the necessary deformation is reached. The algorithm is realized in the computer code written in C++.

1. Introduction

The algorithm for generation of structured grids in volumes of revolution which are subject to deformation is suggested. Considered volumes are obtained by the rotation around the axis through 180° of a plane generatrix curve consisting of straight line segments and arcs of circles. We consider the pressure caused by volumes of revolution (deforming body) on the another volume of revolution (main body). As the result of such application the main body is deformed: the boundary of the main body is moved inside the body and takes the shape of the additional (deforming) body in the region of deformation while the shape of the other part of the main body (which is not subject to deformation) does not change. Since in the general case, the body of revolution is formed by conical, cylindrical and spherical surfaces, the consideration of a pressure by means of a conus, a cylinder and a sphere onto the main body is the necessary stage for elaborating the algorithm for the cases of pressure by means of a general body of revolution onto the general body of revolution. Such cases are considered first. In the paper, further experience of the development of the algorithm for deformation by the more general bodies of revolution is described. Some examples are given.

Grids in deformed volumes are used for mathematical modeling the processes of multi-material hydrodynamics [1]. The general approach used for grid generation is the mapping approach [2]. According to this approach constructing a structured grid in the domain of a complicated geometrical form called the physical domain is carried out by means of a continuous mapping of the domain of a simpler form (in our case a rectangular parallelepiped) called the computational domain [3]. Thus, the physical domain in this way of grid construction is represented as a curvilinear hexahedron and the computational grid is the image of a uniform rectangular grid in the rectangular parallelepiped under the considered mapping and consists of images of unit cubes which are hexahedral cells. The way of representation of a domain as a curvilinear hexahedron defines the configuration of the domain and the structured grid.

Within the mapping approach a method for generation of structured grids in the volumes of revolution [4, 5] has been developed using the variational technique [6] which allows to construct grids of desired quality, nondegenerate [3, 7, 8] closed to uniform and orthogonal ones and, if necessary, adaptive to the solution of the physical problem [6]. It would be attractive to apply the method developed for the volume of revolution for the case of the deformed volume of revolution.

In this connection, it is obvious the idea to elaborate the non-stationary algorithm for generation of moving grids changing the boundary of the volume of revolution according to the deformation process. The algorithm represents the iterative procedure at each step of which the boundary of the main body is deformed under the pressure of the additional body (additional body moves inside the main body), boundary grid nodes of the main body remove on the deforming surface and then optimization of the deformed grid is carried out by the variational technique [4, 5, 6].

The problems of constructing moving grids are described by many authors (see, for example, a general review and underlying principles in [9] and a review, some examples and a comparison of different mesh morphing methods applied for the case of 3D shape optimization in [10]). The main difficulty of non-stationary algorithms (see [11]) is to provide the generation of a grid satisfying the given quality criteria (nondegeneracy, closeness to uniformity, orthogonality and etc. [5, 6]) automatically and without any interference of the user at each step of the process. So, the described algorithm is a solution of a complicated problem and it represents a new computational technique for multimaterial hydrodynamic simulation and for solving other significant scientific and engineering problems.

The considered problem of grid generation in the volume of revolution deformed by the another volume of revolution is formulated in section 2. In section 3, the non-stationary grid generation algorithm is described. Examples of calculations of grids are given in section 4.

2. Formulation of the problem

In the Cartesian coordinate systems, X with axis $\{x^1, x^2, x^3\}$ (main system) and Ξ with the axis $\{\xi^1, \xi^2, \xi^3\}$ (additional system), the volumes (bodies) of revolution, the main one G_x ((Fig. 1(b))) and the additional one Q_ξ (Fig. 1(a)), are given correspondingly. Each of the volumes is obtained by the the rotation of the plane generatrix curve consisting of straight line segments and arcs of circles (elements) through the angle $\varphi = \pi$ around the axis. The generatrix of the main volume is given in the plane $\{x^1, x^3\}$, and the generatrix of the additional volume is given in the plane $\{\xi^1, \xi^3\}$. The rotation is carried out the around axis x^3 for the main body and around the axis ξ^3 for the additional body.

The origin $O_{\Xi X} = (x^1_o, x^2_o, x^3_o)$ of the coordinate system Ξ in the coordinate system X and the matrix C of the transformation from the coordinate system Ξ to the coordinate system X are given. The vector of deformation $\vec{V}_X$ in the coordinate system X which shows the direction of the deforming body pressure onto the main body is also given. The origin $O_{\Xi X}$ and the matrix C are given in such a way that after the transformation of coordinates (to the coordinate system

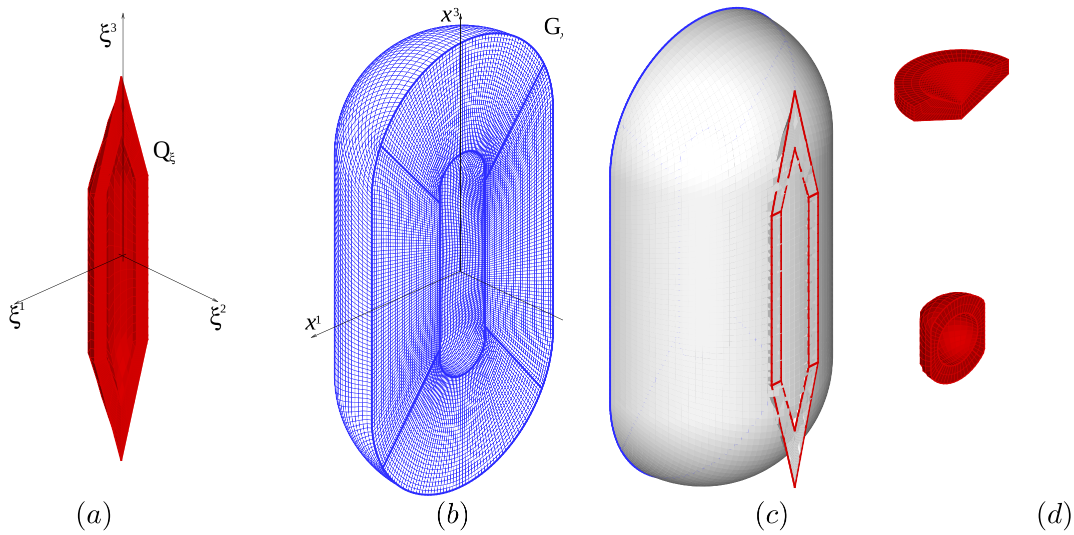

Figure 1. The generation of the deformed grid: (a) domain Q_ξ and (b) grid in the domain G_x, (c) domains Q_x and G_x in the coordinate system X, (d) additional domains Q_ξ.

X) the deforming body Q_x defines its final position inside the main body (Fig. 1(c)).

The structured three-dimensional grid $\mathbf{x}_{ijk} = (x^1_{ijk}, x^2_{ijk}, x^3_{ijk})$, $i = 0, .., N-1$, $j = 0, .., M-1$, $k = 0, .., L-1$ where integers N, M, L give the number of grid nodes is constructed in the domain G_x by the algorithms [4, 5]. The main body corresponding to the domain G_x is deformed by pressure of the additional body. The problem consists in constructing the structured grid in the deformed domain $D = G_x \setminus (G_x \cap Q_x)$.

3. Non-stationary grid generation algorithm

The given shape of the main body is deformed little by little in the iterative process until the deformation reaches the necessary degree (the deforming body turns out in its final position, see Fig. 1(c)). The algorithm consists of three stages. At the first (auxiliary) stage, the parameters (the number and the form of the elements of generatrix) of the deforming body are defined and the movement of the deforming body out of the main body is carried out. The second stage is the deformation of a grid in the main body. At the third stage, the grid is optimized in the correspondence with given quality criteria by the optimization algorithm 3.3. The second and the third stages are continued up to that moment when the additional body reaches its final position (see Fig. 1(c)).

3.1. Definition of parameters

By the text files with the description of the geometries of the main and additional bodies, we define the parameters of the considered case. Each element of the generatrix (a straight line segment or an arc of a circle) defines the form of the corresponding surface of revolution (conical, cylindrical or spherical) which further defines the way of projection of nodes on this surface. Then, we prepare for the process of deformation. During the process of deformation the deforming body has to move inside the main body in the direction given by the vector of deformation. For simplicity of computations, we move not the additional body but the main body and perform all computations in the auxiliary coordinate system Ξ. First, we move the main body out the additional body. The touch is allowed. Then we begin to move the main body. Thus, the coordinates of deforming surface of the auxiliary body do not change in the process of deformation, instead of this the main body moves.

3.2. Deformation of a grid
At this stage, such step of deformation (movement of the main body) is chosen that deformation does not affect (the additional body after movement of the main body does not touch) inner nodes of a grid of the main body. The boundary nodes of the main body subjected to deformation (got inside the additional body) remove on the deforming surface of the additional body: these nodes are projected ([12, 13]) on this surface. Here, it is defined the element of the generatrix forming the corresponding surface of revolution at which the nodes are projected. Then we perform the back transformation to the main coordinate system X and carry out the optimization stage.

3.3. Optimization of a grid
Initial [4] and deformed grids often contain cells unsatisfactory in their quality and differ very much in shape and size and close to degenerate. To construct the grid satisfying the users by its quality from the deformed grid, the optimization [5] or global conservative reconstruction (remapping) of a grid is applied. The conservative property is understood as the property to conserve the volume both locally and globally in the whole domain. The reconstruction is carried out by the variational method [5] via minimization of the discrete functional

$$D = D_{\text{u}} + A_{\text{o}} D_{\text{o}} \tag{1}$$

where $A_{\text{o}} > 0$ is a weight coefficient regulating the closeness of a grid to orthogonal one, D_{u} and D_{o} are measures of closeness of a grid to a uniform one and an orthogonal one, correspondingly:

$$D_{\text{u}} = \sum_{ijk} \Bigg\{ \left[r_{i+1,j,k} - r_{i-1,j,k}\right]^2 \left(\frac{1}{r^2_{i+1,j,k}} + \frac{1}{r^2_{i-1,j,k}} \right)$$

$$+ \left[r_{i,j+1,k} - r_{i,j-1,k}\right]^2 \left(\frac{1}{r^2_{i,j+1,k}} + \frac{1}{r^2_{i,j-1,k}} \right) + \left[r_{i,j,k+1} - r_{i,j,k-1}\right]^2 \left(\frac{1}{r^2_{i,j,k+1}} + \frac{1}{r^2_{i,j,k-1}} \right) \Bigg\},$$

$$D_{\text{o}} = \sum_{ijk} \sum_{p=1}^{4} \left(\frac{1}{\sin^2 \varphi^p_{ij}} + \frac{1}{\sin^2 \varphi^p_{ik}} + \frac{1}{\sin^2 \varphi^p_{jk}} \right).$$

Here $r_{i\pm1,j,k} = |\mathbf{h}_{i\pm1}| = |\mathbf{x}_{i\pm1,j,k} - \mathbf{x}_{ijk}|$. Analogously, $r_{i,j\pm1,k} = |\mathbf{h}_{j\pm1}|, r_{i,j,k\pm1} = |\mathbf{h}_{k\pm1}|$. Values φ^p_{ij} $(p = 1, 2, 3, 4)$ denote the angles between vectors $\mathbf{h}_{i\pm1}$ and $\mathbf{h}_{j\pm1}$ (see. Fig. 2). Summation is carried out over all inner nodes of the grid.

In [5] the continuous analogue of this functional is supplied and different variational problems are considered. The distinctive feature of the functional of optimality (1) is a special way of a uniformity criterion formalization. It defines the type of Euler equations for constructing grid (hyperbolic in the wide sense), permits to consider different types of boundary conditions in the variational problems for grid generation (fixed and free nodes, orthogonality of coordinate lines to the boundary) and provides good computational properties of grids (see [5, 6]). The construction of the functional of orthogonality D_{o} allows to refer the method to barrier methods [14], which possess a “barrier” for degenerate elements (for details, see also [5]).

In [5], during optimization when boundary grid nodes are considered to be free the movement of nodes is carried out along the surface composed of ruled faces of grid cells. For this movement grid nodes are close to the surface of revolution but do not belong to it. This can lead to the

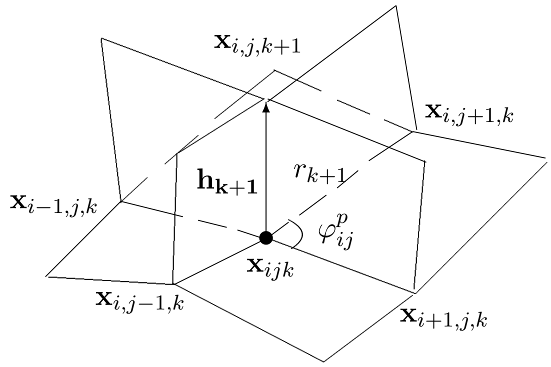

Figure 2. The elements of cells of a three-dimensional grid.

lost of the volume of the domain and the accuracy in the approximation of boundary conditions, so the algorithm for correction of grid nodes to the surfaces of revolution [12, 13, 15, 16] has been developed. Its main idea is in the following. The correcting node is projected on the corresponding surface of revolution. Deformed nodes are projected on the surfaces of revolution of the additional body, others (arranged on the ruled surfaces of cell faces of the main body) on the surfaces of revolution of the main body. In the last case what a surface of revolution has to be chosen is defined by the third coordinate of the node. Each element of the generatrix curve has its own interval of the change of this coordinate. For each face the numbers of elements generating the face are specified. What interval includes the third coordinate of the node defines the number of the element of the generatrix curve generating the corresponding surface for which projection has to be carried out. Thanks to this algorithm the movement of grid nodes can be carried out along surfaces of revolution and the reconstruction process can be conservative.

As a rule only a slight part of the domain is subject to deformation therefore it is not worthwhile to optimize the grid in the whole domain at each step of deformation. It is sufficient to optimize the fragment of a grid which is subjected to deformation.

4. Examples of calculations

The calculations are carried out on the personal computer (quad-core processor Intel Core i5-4570, 3.2 GHz, 4Gb of RAM, 64-bit OS). In Fig. 3 examples of grids for the main volume shown in Fig. 1(b) deformed by the additional volumes (Fig. 1(a), (d)) are given. The grid in these volumes contains 81x81x30=196830 nodes. In dependence of the given parameters (the weight of orthogonality, the numbers of iteration of deformation and optimization of a grid, and the degree of deformation) the calculations can be lasted from some minutes up to some hours. For optimization of a grid in the deformed fragment, the time of calculation reduces several times. Testing of grids is carried out according to criteria [8]. According to [8] cells degenerating into prisms (admissible in [1]) arise along edges of joining faces lying in one plane, other cells are nondegenerate hexahedral cells [7]. Among the last ones there are no twisted hexahedral cells exotic in shape. The closeness of grids to uniform and orthogonal one is estimated by the values of functionals $D_{\mathrm{u}}, D_{\mathrm{o}}$ [6].

Conclusion

Described algorithms are realized in computer codes written in C++. Further development of the technology will be performed for the cases of deformation of the volume of revolution by the more complicated volumes of revolution.

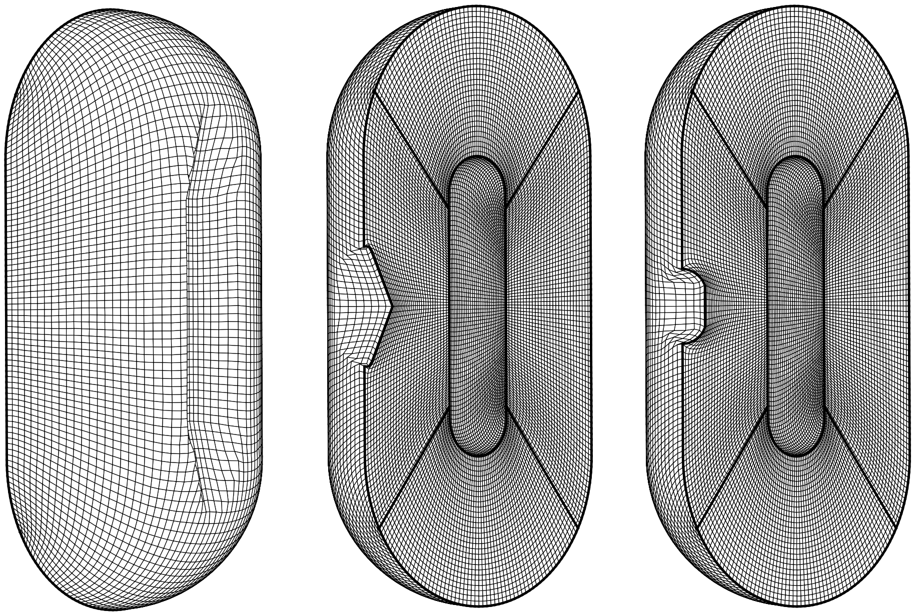

Figure 3. Examples of grids in deformed volumes.

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